Dynamical Sampling - Linear Algebra and an excursion into Control Theory

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Sampling is about recovering all values of a given signal (function) from the knowledge of some (in general few) values (samples). A famous theorem from Shannon (Whitacker or even Cauchy) states that in order to recover a *bandlimited* function of say $|\Omega|$ bandwith, we need to sample at least at all points that are multiples of $\frac{1}{|\Omega|}k$, with k integer numbers. In practice, that means to place sensors at all these points and obtain measurements (samples) to reconstruct the function.

Dynamical Sampling is about recovering the signal f from fewer samples than the ones required by the Theorem of Shannon, in the case that the vector *evolves* through the action of an operator T.

The question we adress is: is it still possible to recover f if we know the samples at certain points, but in addition we know the samples of Tf , TTf , ..., $TTT\cdots Tf$ at these points.

In the finite dimensional setting (the Hilbert space is $\mathbb{R}^n$ or $\mathbb{C}^n$) this reduces to a problem of linear algebra.

The infinite dimensional situation is much more delicate - and has only been solved completely recently using deep results from Complex Analysis.

The generalization to the continuous setting, uses the notion of *continuous frames* and surprisingly the discrete and the continuous case are actually quite similar, and we therefore recover the known theorems from *classical* dynamical sampling for the continuous case. Moreover, relating the problem of Dynamical Sampling to the problem of Observability in Control Theory, we can obtain new results in both settings.

References

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