Title: Optimization in the construction of multidimensional wavelets

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Abstract

The multiresolution analysis (MRA) is a theoretically systematic way of constructing orthonormal wavelet bases for $L^2(\mathbb{R}^n, \mathbb{C})$. Relying heavily on complex analysis, Daubechies used the MRA to derive a special class of compactly supported smooth orthogonal wavelets on the line. Alternatively, Franklin, Hogan, and Tam developed techniques which have been successful in reproducing the Daubechies' wavelets using optimization and projection methods, effectively bypassing the role of complex analysis. The same techniques have been applied to produce nonseparable examples of compactly supported smooth orthogonal wavelets on the plane.

In this talk, we discuss the extensions of these constructions to allow for the optimization of wavelets' cardinality and symmetry. Additionally, we present an alternative approach to achieve cardinality and symmetry on the line by solving an optimization problem arising from a quadrature mirror filter construction using the Zak transform. Finally, we discuss the possibility of extending these techniques to construct quaternionic wavelets.