stability and convergence of Galerkin discretizations of the Helmholtz equation

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We consider boundary value problems for the Helmholtz equation at large wave numbers k. In order to understand how the wave number k affects the convergence properties of discretizations of such problems, we develop a regularity theory for the Helmholtz equation that is explicit in k. At the heart of our analysis is the decomposition of solutions into two components: the first component is an analytic, but highly oscillatory function and the second one has finite regularity but features wavenumber-independent bounds.

This new understanding of the solution structure opens the door to the analysis of discretizations of the Helmholtz equation that are explicit in their dependence on the wavenumber k. As a first example, we show for both a conforming high order finite element method (hp-FEM) as well as the high order discontinuous Galerkin FEM (hp-DGFEM) that quasi-optimality is guaranteed if (a) the approximation order p is selected as $p = O(\log k)$ and (b) the mesh size h is such that kh/p is small.

As a second example, we consider combined field boundary integral equation arising in acoustic scattering. Also for this example, the same scale resolution conditions as in the high order finite element case suffice to ensure quasi-optimality of the Galekrin discretization.