Fast Galerkin Methods for Parabolic Boundary Integral Equations

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It is well known that the layer potentials of the heat equation are coercive in appropriate anisotropic Sobolev spaces which implies stability and error estimates for Galerkin methods. The resulting linear systems are block-lower triangular and can be solved by block-forward elimination. To handle the cost of dense matrix calculation a space-time version of the fast multipole method is used, which allows the computation a matrix vector product with nearly optimal cost. Further, if the space-time meshwidths satisfy $h_t \leq Ch_x^2$ the conditioning of the linear system in each time step does not grow with mesh refinements.

We also discuss the application of the methodology to three-dimensional transient Stokes flow. Perhaps surprisingly, this is not straight forward, because of different properties of the fundamental solutions of the heat and Stokes equations.